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The Mellin moments of deep inelastic structure functions at two loops

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We perform the analytic calculation of the Mellin moments of the structure functions F_2 and F_L in perturbative QCD up to second order corrections and in leading twist approximation. We calculate the 2-loop contributions to the anomalous dimensions of the singlet and non-singlet operator matrix elements and the 2-loop coefficient functions of F_2 and F_L. Our results are in agreement with earlier calculations in the literature.

1. INTRODUCTION

Deep inelastic lepton-hadron scattering is one of the best studied reactions today. It provides unique information about the structure of the hadrons and tests one of the most important predictions of perturbative QCD, the scale evolution of the structure functions [1,2,3]. At the same time, the ever increasing accuracy of deep inelastic scattering (DIS) experiments demands more accurate theoretical predictions and despite all achievements [4,5,6,7], no complete next-to-next-to-leading order (NNLO) analysis for DIS reactions is available.

The necessary perturbative QCD predictions at 3-loops for the anomalous dimensions of the structure functions and the structure function F_L entering in the ratio $R = \sigma_L/\sigma_T$, are still unknown, except for a number of fixed Mellin moments of F_2 and F_L [8,9]. These have already been used in NNLO analyses [9,10,11]. In general however, this limited information about some fixed Mellin moments is not sufficient to allow for NNLO analyses of all data of DIS experiments and as a consequence, there are still considerable uncertainties on the parton densities, e.g. on the gluon density at small x.

Progress beyond the state of the art has to explore different directions, since a straightforward extension of ref.[9] to calculate more fixed Mellin moments, is not feasible. One such possibility that actually dates back to the origins of QCD [1,3], is to calculate the Mellin moments of the structure functions analytically as a general function of N. This approach, further pioneered by Kazakov and Kotikov [4] to obtain the structure function F_L at 2-loops, turns out to be flexible and very promising in view of the ultimate goal, the anomalous dimensions at 3-loops. As a first step and to demonstrate the power of the method, we report here on the recalculation [12] of the perturbative QCD contributions up to 2-loops to the structure functions F_2 and F_L.

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2. FORMALISM

The optical theorem relates the DIS structure functions to the forward scattering amplitude of photon-nucleon scattering, $T_{\mu\nu}$, which has a time-ordered product of two local electromagnetic currents, $j_\mu(x)$ and $j_\nu(z)$, Fourier transformed into momentum space, to which standard perturbation theory applies. The operator product expansion (OPE) allows to expand this current product around the lightcone $(x-z)^2 \sim 0$ into a series of local composite operators of leading twist and spin N. The anomalous dimensions of matrix elements of these operators govern the scale evolution of the structure functions, while the coefficient functions multiplying these matrix elements are calculable order by order in perturbative QCD.

In this way the Mellin moments of DIS structure functions can naturally be written in the parameters of the OPE,

$$\frac{1+(-1)^N}{2} \int_0^1 dx\, x^{N-2} F_i(x,Q^2) = \sum_{k=\mathrm{ns,q,g}} C^k_{i,N}\left(Q^2/\mu^2, \alpha_s\right) A^k_{\mathrm{nucl},N}(\mu^2)\,, \qquad (1)$$

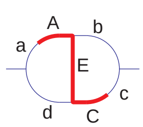
where $i = 2, L$ and all even Mellin moments are

fixed. $A_{\rm nucl}$ denote the spin-averaged hadronic matrix elements and C_i the coefficient functions and the sum extends over the flavour non-singlet and singlet quark and gluon contributions.

The OPE is an operator statement and therefore independent of a particular hadronic matrix element, so that it is standard to calculate partonic structure functions with external quarks and gluons in infrared regulated perturbation theory. In practice, this procedure reduces to the task of calculating the N-th moment of all 4-point diagrams that contribute to $T_{\mu\nu}$ at a given order in perturbation theory. This will be shown in more detail below.

3. METHOD

To illustrate the method of calculating Mellin moments of DIS structure functions, consider the following diagram, which is the most complex topology at 2-loops, since three propagators with P-momentum are present.



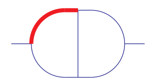
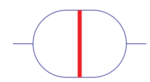
$$= \int d^Dp_1 d^Dp_2 \frac{1}{(p_1^2)^a((P+p_1)^2)^A} \quad (2)$$

$$\times \frac{1}{(p_2^2)^b(p_3^2)^c((P-p_3)^2)^C(p_4^2)^d((P+p_5)^2)^E} \,.$$

The fat lines denote P-dependent propagators with $P^2=0$, while the Q-momentum flows from right to left through the diagram. Usually, there are only single powers of the propagators, $a = b = ... = 1$ in eq.(2).

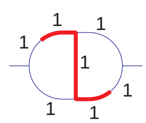
We wish to compute the N-th moment of this diagram. In a naive approach one would expand all P-dependent propagators into sums over $P{\cdot}p_i/p_i^2$ using $P^2=0$. Then scaling arguments require the final answer for the N-th moment to be proportional to $(P{\cdot}Q/Q^2)^N$ and one is left to calculate 2-point functions with symbolic powers of scalar products in the numerator and denominator. However, this procedure leads to multiple nested sums, which in general are very difficult to evaluate.

More sophisticated ideas have been used in ref.[4]. There it is shown how to determine reduction identities for a given diagram in dimensional regularization in such a way, that is possible to set up recursion relations in the number of moments N. A systematic classification of all 2-loop topologies reveals two basic building blocks. The two diagrams, each with only one P-dependent propagator,

 ,  , (3)

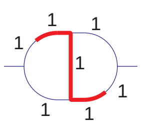
are shown in diagrammatic notation similar to eq.(2). These diagrams can be calculated at the cost of one sum over Γ-functions. In dimensional regularization, $D = 4-2\epsilon$, the Γ-functions can be expanded in ϵ and the sum can be solved to any order in ϵ.

The other more complex topologies like eq.(2) can be expressed in sums over these basic topologies by means of recursions, so that a complex diagram is broken up systematically into its building blocks. For example, the simple and beautiful recursion relation for eq.(2) reads



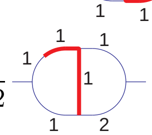
$$\left(\frac{2P{\cdot}Q}{Q{\cdot}Q}\right)^k = \quad (4)$$

$$-\frac{N-k-D+6}{N-k+2}$$



$$\left(\frac{2P{\cdot}Q}{Q{\cdot}Q}\right)^{k+1}$$

$$+\frac{2}{N-k+2}$$



$$\left(\frac{2P{\cdot}Q}{Q{\cdot}Q}\right)^k ,$$

which relates the N-th moment of this diagram to a diagram of a simpler structure, where the P-dependent propagator $(P{-}p_3)^2$ has been eliminated. The recursion eq.(4) can be solved and for the second diagram on the right hand side similar relations are derived.

Amazingly enough, if the recursion relations are set up in this way, all the multiple nested sums can all be solved successively in terms of harmonic sums, the basic functions of weight m being defined as follows [13,14]

$$S_m(N) = \sum_{i=1}^{N} \frac{1}{i^m}\,, \quad S_{-m}(N) = \sum_{i=1}^{N} \frac{(-1)^m}{i^m}\,, \quad (5)$$

while higher functions can be defined recursively

$$S_{m_1,\ldots,m_k}(N) = \sum_{i=1}^{N} \frac{1}{i^{m_1}} S_{m_2,\ldots,m_k}(i)\,, \qquad (6)$$

$$S_{-m_1,\ldots,m_k}(N) = \sum_{i=1}^{N} \frac{(-1)^{m_1}}{i^{m_1}} S_{m_2,\ldots,m_k}(i)\,. \quad (7)$$

Of course, this procedure requires great care in the way the reduction identies are applied. It also relies crucially on all algebraic relations for harmonic sums, which allow to solve the nested sums algorithmically [13] and to express the result in the basis of harmonic sums eqs.(5)–(7).

4. RESULTS

The analytic calculation has been done with the symbolic manipulation program FORM [15]. All recursion relations have been implemented in a program, that reduces diagrams for DIS structure functions up to 2-loops to multiple nested sums over the basic building blocks and subsequently calls the SUMMER [13] algorithm, to solve these nested sums in terms of the basis of harmonic sums. The database of diagrams, identical to the one used in ref.[9], was generated with the help of QGRAF [16]. The program has been optimized by tabulating basic building blocks, so that the calculation of all 425 diagrams for F_2 and F_L up to 2-loops is done in a few of hours on a Pentium Pro PC.

The calculation is performed in dimensional regularization, $D = 4 - 2\epsilon$, and renormalization and mass factorization proceeds as described in refs.[6,7,8,9]. To extract the gluon anomalous dimension at 2-loops, we also calculate the unphysical structure functions of a scalar particle ϕ, that couples to gluons only [9]. These provide us with the necessary renormalization constants of the singlet operators in eq.(1) and can be obtained from an additional operator $\phi F^{a\,\mu\nu} F^{a}_{\mu\nu}$ in the Lagrangian.

Finally, we obtain the even Mellin moments of the complete set of anomalous dimensions, the flavour non-singlet and singlet quark and the gluon coefficent functions up to 2-loops. Our results for the even moments of the anomalous dimensions agree with the ones published in refs.[1,3], while our results for the coefficient functions agree with those of refs.[2,6,7,8,9].

We have also calculated all even moments of the $\mathcal{O}(\epsilon)$-terms of all structure functions at 2-loops, needed to extract the coefficient functions at 3-loops after mass factorization. In the appendix, we present our result for the even moments of the pure singlet quark coefficent function $c_2^{\rm ps}$, which has not been shown in this form before. The complete results and more details of the calculation will be published elsewhere [12].

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APPENDIX

The even Mellin moments of the pure singlet quark coefficient function $c_2^{\rm ps}$ at 2-loops.

$$
\begin{aligned}
c_2^{\rm ps} = n_f C_F \Big[& \qquad\qquad\qquad\qquad\qquad (8)\\
& -133/81\;\delta(N-2)\; + \theta(N-3)\;\Big\{\\
& -344/27 S_1(N-2) - 130/27 S_1(N-1)\\
& -1714/27 S_1(N+1) + 448/27 S_1(N+2)\\
& +580/9 S_1(N) - 16/3 S_{1,-2}(N-2)\\
& +64/3 S_{1,-2}(N-1) + 64/3 S_{1,-2}(N+1)\\
& -16/3 S_{1,-2}(N+2) - 32 S_{1,-2}(N)\\
& +104/9 S_{1,1}(N-2) - 416/9 S_{1,1}(N-1)\\
& -272/9 S_{1,1}(N+1) + 32/9 S_{1,1}(N+2)\\
& +184/3 S_{1,1}(N) - 16/3 S_{1,1,1}(N-2)\\
& +4/3 S_{1,1,1}(N-1) + 4/3 S_{1,1,1}(N+1)\\
& -16/3 S_{1,1,1}(N+2) + 8 S_{1,1,1}(N)\\
& +16/3 S_{1,2}(N-2) - 4/3 S_{1,2}(N-1)\\
& -4/3 S_{1,2}(N+1) + 16/3 S_{1,2}(N+2)\\
& -8 S_{1,2}(N) + 56 S_2(N-1)\\
& +136/9 S_2(N+1) + 128/9 S_2(N+2)\\
& -256/3 S_2(N) - 16 S_{2,1}(N+1)\\
& +16 S_{2,1}(N+2) + 8 S_{2,1,1}(N-1)\\
& -8 S_{2,1,1}(N+1) - 8 S_{2,2}(N-1)\\
& +8 S_{2,2}(N+1) + 2 S_3(N-1)\\
& +154/3 S_3(N+1) - 64/3 S_3(N+2)
\end{aligned}
$$

$$
-32S_3(N) - 16S_{3,1}(N-1) \\
+16S_{3,1}(N+1) + 20S_4(N-1) \\
-20S_4(N+1)\}\Big] \,.
$$

REFERENCES

[1] D.J. Gross and F. Wilczek, Phys. Rev. **D8** (1973) 3633.

[2] W.A. Bardeen, A.J. Buras, D. Duke and T. Muta, Phys. Rev. **D18** (1978) 3998.

[3] E.G. Floratos, D.A. Ross and C.T. Sachrajda, Nucl. Phys. **B129** (1977) 66, ibid. **B139** (1978) 545 (Erratum); idib. **B152** (1979) 493.
A. Gonzalez-Arroyo, C. Lopez and F.J. Ynduráin, Nucl. Phys. **B153** (1979) 161.
G. Curci, W. Furmanski and R. Petronzio, Nucl. Phys. **B175** (1980) 27.
W. Furmanski and R. Petronzio, Phys. Lett. **B97** (1980) 437.

[4] D.I. Kazakov and A.V. Kotikov, Nucl. Phys. **B307** (1988) 721, ibid. **B345** (1980) 299 (Erratum).

[5] D.W. Duke, J.D. Kimel and G.A. Sowell, Phys. Rev. **D25** (1982) 71.
A. Devoto, D.W. Duke, J.D. Kimel and G.A. Sowell, Phys. Rev. **D30** (1984) 541.
D.I. Kazakov, A.V. Kotikov, G. Parente, O.A. Sampayo and J. Sanchez Guillen, Phys. Rev. Lett. **65** (1990) 1535.

[6] S.A. Larin and J.A.M. Vermaseren, Z. Phys. **C57** (1993) 93.

[7] E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. **B383** (1992) 525.
E.B. Zijlstra, PhD thesis, RX1449 Leiden (1993).

[8] S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, Nucl. Phys. **B427** (1994) 41.

[9] S.A. Larin, P. Nogueira, T. van Ritbergen and J.A.M. Vermaseren, Nucl. Phys. **B492** (1997) 338.

[10] A.L. Kataev, A.V. Kotikov, G. Parente and A.V. Sidorov, Phys. Lett. **B417** (1998) 374.

[11] J. Santiago and F.J. Ynduráin, these proceedings, `hep-ph/9907387`; `hep-ph/9904344`.

[12] S. Moch and J.A.M. Vermaseren, to be published.

[13] J.A.M. Vermaseren, Int. J. Mod. Phys. **A14** (1999) 2037.

[14] J. Blümlein and S. Kurth, Phys.Rev. **D60** (1999) 014018.

[15] J.A.M. Vermaseren, Symbolic Manipulation with FORM, published by CAN (Computer Algebra Nederland), Kruislaan 413, 1098 SJ Amsterdam, 1991, ISBN 90-74116-01-9.

[16] P. Nogueira, J. Comput. Phys. **105** (1993) 279.